

NAG Toolbox for MATLAB

s09aa

1 Purpose

s09aa returns the value of the inverse circular sine, $\arcsin x$, via the function name. The value is in the principal range $(-\pi/2, \pi/2)$.

2 Syntax

```
[result, ifail] = s09aa(x)
```

3 Description

s09aa calculates an approximate value for the inverse circular sine, $\arcsin x$. It is based on the Chebyshev expansion

$$\arcsin x = x \times y(x) = x \sum_{r=0}^{\infty} a_r T_r(t)$$

where $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$ and $t = 4x^2 - 1$.

For $x^2 \leq \frac{1}{2}$, $\arcsin x = x \times y(x)$.

For $\frac{1}{2} < x^2 \leq 1$, $\arcsin x = \text{sign } x \left\{ \frac{\pi}{2} - \arcsin \sqrt{1 - x^2} \right\}$.

For $x^2 > 1$, $\arcsin x$ is undefined and the function fails.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: $|x| \leq 1.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail** – **int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

The function has been called with an argument greater than 1.0 in absolute value; $\arcsin x$ is undefined and the function returns zero.

7 Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\arcsin x \sqrt{1-x^2}} \times \delta \right|.$$

That is, a relative error in the argument x is amplified by at least a factor $\frac{x}{\arcsin x \sqrt{1-x^2}}$ in the result.

The equality should hold if δ is greater than the *machine precision* (δ is a result of data errors etc.) but if δ is produced simply by round-off error in the machine it is possible that rounding in internal calculations may lose an extra figure in the result.

This factor stays close to one except near $|x| = 1$ where its behaviour is shown in the following graph.

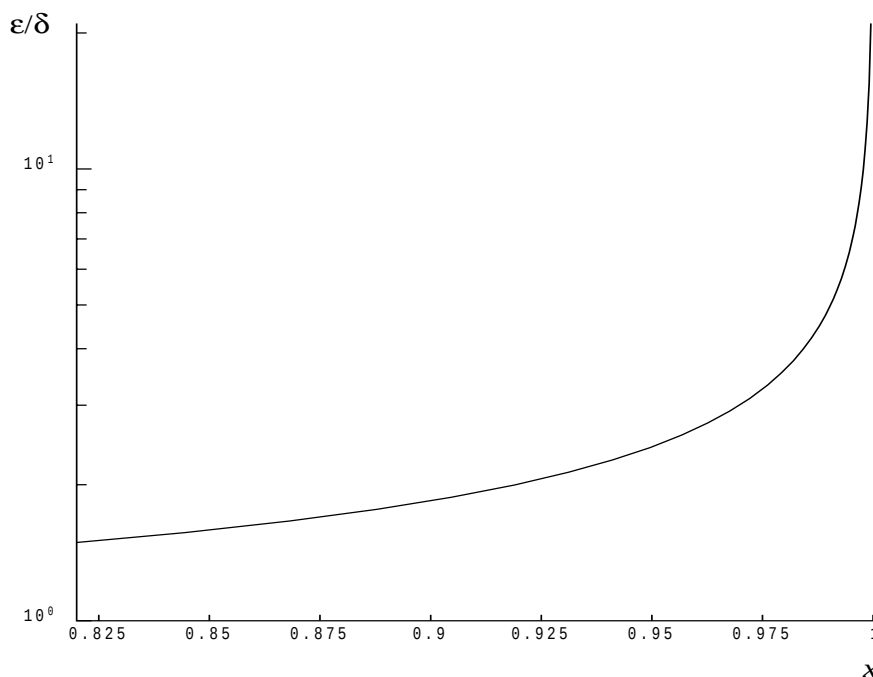


Figure 1

For $|x|$ close to unity, $1 - |x| \sim \delta$, the above analysis is no longer applicable owing to the fact that both argument and result are subject to finite bounds, ($|x| \leq 1$ and $|\arcsin x| \leq \frac{1}{2}\pi$). In this region $\epsilon \sim \sqrt{\delta}$; that is the result will have approximately half as many correct significant figures as the argument.

For $|x| = 1$ the result will be correct to full *machine precision*.

8 Further Comments

None.

9 Example

```
x = -0.5;  
[result, ifail] = s09aa(x)  
  
result =  
    -0.5236  
ifail =  
         0
```
